1315001 Boston U., Mass.

Boston Univ.

12p.

UNPUBLISHED PRELIMINARY DATA

RESEARCH REPORT NO. 6, ~~September 1963, NASA GRANT G246-62~~ NsG

INTERSECTIONS OF LUNAR CRATERS

N64 13928* refs CODE-1

[2] (NASA Grant NsG-246-62)

David Friesen Sep. 1963 12p

(NASA CR-55274)

[X] OTS

OTS: $1.60ph, $0.80mf

Introduction

Visual observation of the moon shows that many lunar craters overlap. Parts of the wall of a crater may be broken down, and another crater will intrude into this area. Quick inspection reveals that most of the larger craters (at least in the highland area) are intersected by other craters in this manner.

In this paper an examination of intersection frequency (that is to say, the average number of intersections/craters by smaller craters) will be made for both the large and smaller craters in the highland region. A theoretical expected frequency is calculated and compared with the actual frequency measured from the Boston University Catalog of Lunar Craters (1).

Expected Frequency of Intersection

Lunar craters exhibit a cumulative size distribution of the form

$$N(r) = A\, r^{B} \qquad (1.1)$$

Jhi

where N is the number of craters in a given area with radius $\geq r$, and A, B are constants determined from crater counts. (2,3) Thus the density distribution function is

$$D(r) = \frac{dN(r)}{dr} \qquad (1.2)$$

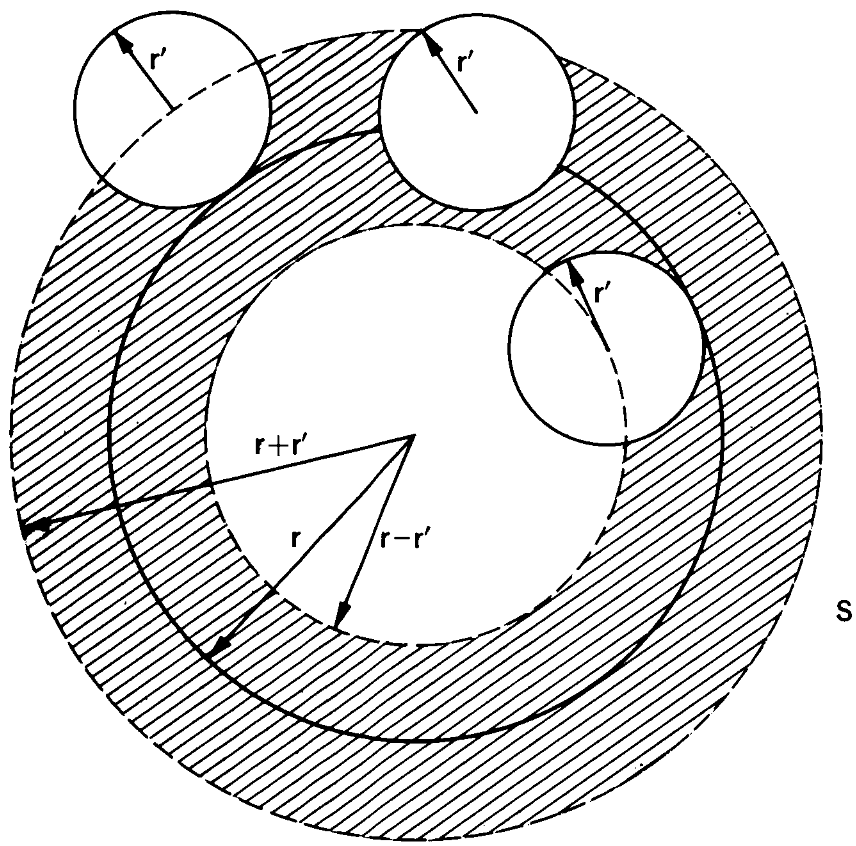
or

$$D(r) = AB\, r^{B-1} \qquad (1.3)$$

with $D(r)dr$ being the number of craters in the size range r to $r + dr$. In the following development the given area will be understood to be $1\ km^2$, and the results will need to be normalized for other values of given area.

If a crater with radius in the interval r to $r + dr$ is to have its bounding wall intersected by a crater of smaller radius in the interval r' to $r' + dr'$ $(r' < r)$, the center of the smaller (r') crater must lie in an annulus about the center of the larger crater. This can be seen in Figure 1, where the annulus is the cross-hatched area. This annulus, or impact area for a single crater of radius r, has an area

$$S = 4\pi r r' \ . \qquad (1.4)$$



Impact Area and Possible Intersection Configurations for Intersection of Crater of Radius r by Craters of Radius r′.

FIGURE 1

The fractional impact area per unit area presented by craters with radius in the range r to $r + dr$ (for intersection by craters of radius $r' < r$) is then

$$I = S \cdot D(r)dr \tag{1.5}$$

or

$$I = 4\pi r' ABr^{B-1}dr \, . \tag{1.6}$$

We assume that the center coordinates of craters are located randomly on the lunar surface. Consider then the number of craters with radius r' in the range r' to $r' + dr'$. There are

$$D(r')dr' = ABr'^{B-1}dr' \, . \tag{1.7}$$

craters in this size range. Since I represents the fractional impact area per unit area the number of intersections per unit area will be

$$D(r')dr' \cdot I = 4\pi A^2B^2r^Br'^Bdrdr' \tag{1.8}$$

Recalling that (1.8) refers to intersections of craters of size r to $r + dr$ by smaller craters of size r' to $r' + dr'$, we consider the total number of intersections of craters of size r to $r + dr$ by craters of smaller size ranging from $r' = \ell$ (some arbitrary lower limit) to $r' = r$. The significance of the choice of ℓ will be discussed below. This number of intersections, M, is obtained by integrating (1.8) with respect to r' from $r' = \ell$ to $r' = r$

$$M(r, \ell) = \int_{r'=\ell}^{r'=r} 4\pi A^2 B^2 r^B dr r'^B dr' = \frac{4\pi A^2 B^2 r^B dr}{B+1} \left(r'^{B+1} \right) \Big|_{\ell}^{r}$$

$$= \frac{4\pi A^2 B^2 r^{2B+1} dr}{B + 1} - \frac{4\pi A^2 B^2 \ell^{B+1} r^B dr}{B + 1} \qquad (1.9)$$

This gives the number of intersections for unit areas of craters in the range r to $r + dr$ by all smaller craters (to an arbitrary lower limit ℓ).

To compare this number directly with observations, we need to select a finite range for r. Letting r vary over some range a_1 to a_2, we integrate (1.9)

$$M(a_1, a_2, \ell) \int_{a_1}^{a_2} M(r, \ell) dr = \frac{4\pi A^2 B^2}{B + 1} \int_{a_1}^{a_2} \left(r^2 B+1 - \ell^{B+1} r^B \right) dr$$

$$= \frac{4\pi A^2 B^2}{B+1} \left[\frac{a_2^{2B+2} - a_1^{2B+2}}{2(B+1)} - \ell \frac{B+1 \left(a_2^{B+1} - a_1^{B+1} \right)}{B+1} \right] \tag{1.10}$$

Equation (1.10) simplifies somewhat to

$$M(a_1, a_2, \ell) = \frac{4\pi A^2 B^2}{(B+1)^2} \left[a_2^{B+1} \left(\frac{a_2}{2}^{B+1} - \ell^{B+1} \right) - a_1^{B+1} \right.$$

$$\left. \left(\frac{a_1}{2}^{B+1} - \ell^{B+1} \right) \right] \tag{1.11}$$

which is in a form convenient for computation.

Examination of equation (1.11) shows that as long as B is more negative than -1, we must set a non-zero value for the limiting lower radius of intersecting craters, ℓ. As ℓ is taken nearer and nearer to zero, the number of intersections tends to infinity for any crater size range a_1 to a_2. This happens even though the total area of all small craters may tend to a finite limit when B is less negative than -2. (2) To avoid this difficulty we confine our discussion to a given size range of interest, where ℓ is the lower bound of that range. Then meaningful numbers of intersections can be obtained by (1.11).

Comparison of Expected and Actual Intersection Frequency

The actual crater intersection frequency has been measured for the area C5A of the Kuiper Photographic Lunar Atlas (4). This area forms section I of the Boston University Catalog of Lunar Craters (I) and is available on punched cards. A program was written for an IBM 1620 which searches the lunar catalog and tabulates the cases of intersection, thus permitting an actual frequency to be arrived at. The output of the program consists of a record of each individual case of intersection plus a record of intersections for each crater. The search program is given in Appendix A, and the summarized results are tabulated below in Table I.

Table I

Radius Range of Craters (Km) a_1 to a_2	Average Number of Intersections per Crater
15 - 45	2.210
10 - 15	1.470
6 - 10	.370
5 - 6	.435
4 - 5	.363
3 - 4	.106
2 - 3	.090

The average number of intersections per crater was computed for the same ranges using equations (1.1) and (1.11). Computations were made for four sets of the constants A, B as shown in Table II. These calculations were done on an IBM 1620 and the program used is included in Appendix A.

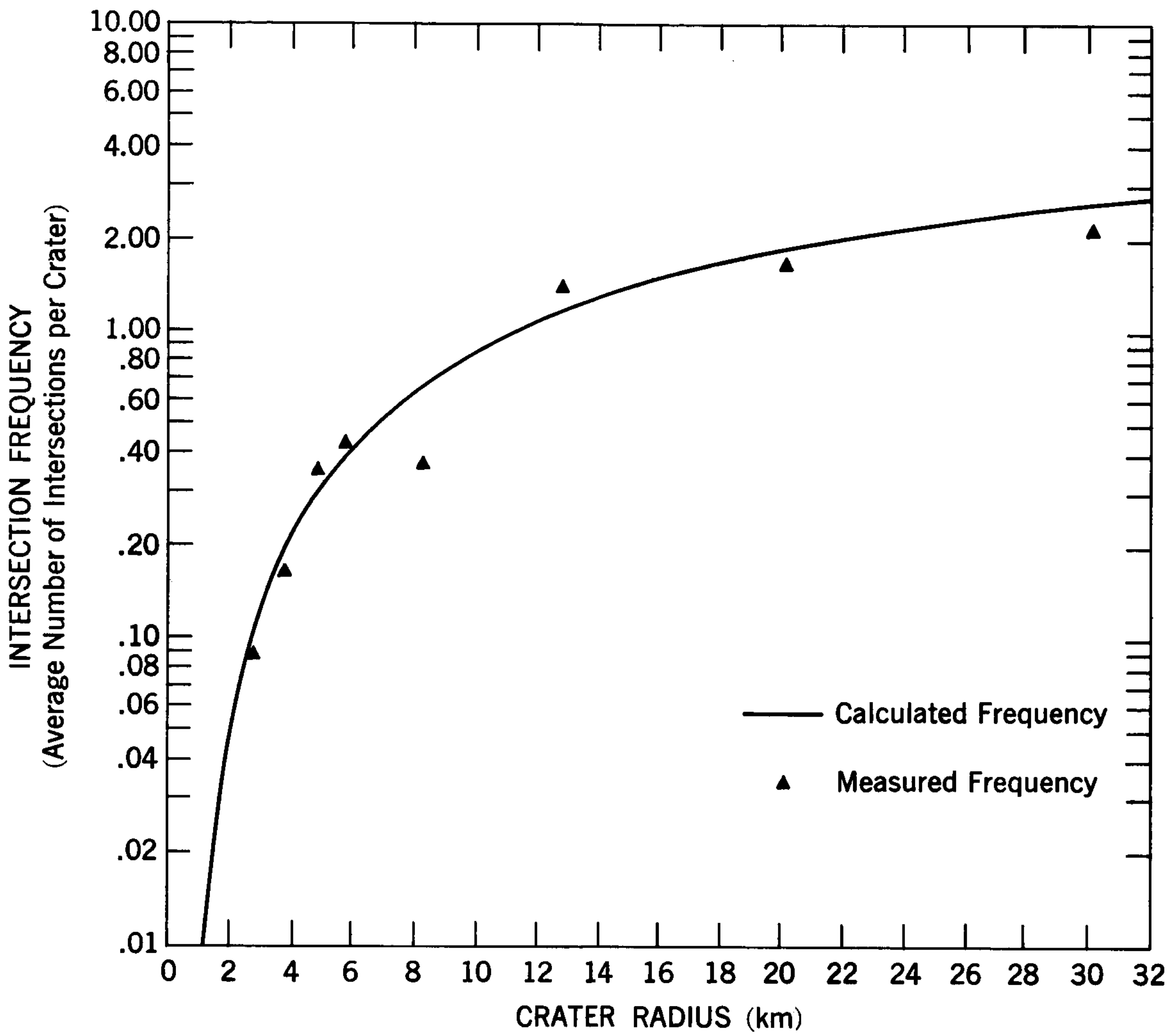
Table II

A	B	Source
4.900×10^3	-1.430	Overall C5 (Mitchell) (1).
8.852×10^3	-1.700	Maurolycus (McGillem & Miller (2))
3.198×10^3	-1.258	Two-Segment C6 (3)
1.590×10^3	-1.334	Two-Segment C5 (3)

The results of the computation are summarized in Table III along with the measured intersection frequency from C5A for comparison.

Table III

Radius Range	Intersection Frequency			
	Computed From			
	C5	Maurolycus	C6	Actual C5
15 - 45	2.605	5.558	2.664	2.210
10 - 15	1.255	2.686	1.129	1.470
6 - 10	.547	1.545	.611	.370
5 - 6	.299	1.025	.380	.435
4 - 5	.188	.780	.280	.363
3 - 4	.140	.537	.185	.166
2 - 3	.093	.300	.098	.090
1 - 2	.011	.070	.022	---



Comparison of Calculated and Actual Intersection Frequency for C5

FIGURE 2

The best agreement with actual intersection frequency is found when the C5 two segment constants A,B are used. (3) A comparison between the actual and computed frequencies for C5 is given in Figure 2.

Conclusions

The close agreement between the calculated intersection frequency and the actual frequency supports the assumptions used in the calculations. Thus it appears that to assume the location of crater centers to be a uniform random function of coordinates is justified. Further, the assumption that the size distribution should be a function of the form

$$N = A\ r^{B}$$

is strengthened.

The best fit between calculations and measurements is obtained for values of A,B considerably different from those previously published for the highland by McGillem and Miller (2). The large overestimation of intersection frequencies found using McGillem and Miller's Maurolycus data suggests that it should be carefully re-examined as being representative of highland crater statistics.

Finally, extrapolation of the derived intersection frequency curves to very small craters shows that intersection is a very rare event at small diameters. Thus for craters of 100 meters and 10 meters radius, we have for intersection frequencies by craters of reasonably similar size (ℓ = 10 meters, 1 meter respectively) the data given in Table IV.

Table IV

Crater Radius R	Intersection frequency For $\ell = 0.1$ R
100 meters	.028
10 meters	.017

Thus intersection of small craters would appear to be rare. This is to be expected, as the intersection frequency depends on the available impact area. The impact area per crater can never be larger than 4 times the crater area, and the area of all very small craters is a small percentage of the total lunar surface area. Taking the low frequency of intersection for small craters and the small area they occupy both into account, it can be expected that the small craters will tend to be isolated from each other. Thus, even though there are very many such small craters, they can be expected to provide little obstruction to movement on the lunar surface.

BIBLIOGRAPHY

(1) Hawkins, Gerald S., and P. W. Mitchell. Catalog of Lunar Craters I and II. Moon Project, Reports 4 and 5, November 1962 and February 1963, NASA G246-62.

(2) McGillem, C. D., and B. P. Miller. Lunar Surface Roughness from Crater Statistics. Journal of Geophysical Research, 67 (12), 4787-4794, Nov. 1962.

(3) Friesen, D. D. Lunar Crater Statistics: The Highland Regions. Moon Project, Report 7, August 1963, NASA G246-62.

(4) Kuiper, G. P., ed. Photographic Lunar Atlas. Univ. of Chicago Press, 1960.